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Phenomenological Issues in TeV scale Gravity with Light Neutrino Masses

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Abstract

The possible existence of bulk singlet neutrinos in the scenario with large compactified dimensions and low string scale M_* has important consequences for low-energy observables. We demonstrate that intergenerational mass splitting and mixing lead to the effective violation of the lepton universality and flavor changing processes in charged lepton sector. Current experimental constraints push M_* to the scale of 10 TeV over most of the interesting range for neutrino mass splitting.

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1 Introduction

Ever since the gauge and matter structure of the Standard Model has fully emerged around the mid 70's Grand Unification [1] has, justifiably, been the guiding light of elementary particle physics. The Grand Unification paradigm has sound support in the experimentally observed data. Most appealing in this regard is in the context of $SO(10)$ unification, where all the Standard Model matter states beautifully fit in fundamental $SO(10)$ representations. It is important to remember that the main motivation for Grand Unification is not the unification of the coupling, but rather the structure of the Standard Model itself. The Grand Unification paradigm, however, indicates that the scale of unification must be of the order 10^{16}GeV . The important facts indicating this large scale are: 1) the longevity of the proton life-time; 2) the qualitatively successful calculation of $\sin^2 \theta_W$; 3) the suppression of the left-handed neutrino masses. It is then further encouraging to learn that the $SO(10)$ unification structure can naturally be embedded in heterotic string models, in which the $SO(10)$ symmetry is broken directly at the string level, thus avoiding some of the difficulties of the field theoretic $SO(10)$ models. This allows for the consistent unification of gravity with the gauge interactions at a scale which is of the order of (or one order above) the GUT scale.

Recently, however, it has been proposed that the fundamental scale of quantum gravity may be as low as the TeV scale, without running into an apparent conflict with the experimental data [2]. It is rather obvious that in this case the Grand Unification structure, like $SO(10)$ unification, must be abandoned. There are many reasons why this must be the case. While it is not unpalatable that the proton decay problem may be circumvented, as we for example can learn from string derived models, Pati-Salam type unification, which qualitatively embodies the general features of $SO(10)$ unification, also predict unification of the top quark and tau neutrino Yukawa couplings, which necessitates the traditional see-saw type mechanism to suppress the left-handed neutrino masses. On the other hand, in the TeV scale gravity scenario the see-saw scale is too low and we can see that the tau-neutrino mass is much above the experimentally preferred region. In this case it is clear that the origin of the right-handed neutrino fields must be entirely different from the other Standard Model states, hence disallowing the underlying $SO(10)$ unification structure. It seems also rather difficult to imagine that the TeV gravity idea can successfully be incorporated in string theory for the following reason. As argued above it is rather evident that any such string model will have to derive the Standard Model gauge group directly at the string level. A general argument in heterotic string theory, which relies on modular invariance, shows that in that case the string spectrum necessarily contains states which carry fractional electric charge [4]. The argument applies to closed string theories, and it is naturally of interest whether it can be extended to type I constructions in which the TeV scale gravity can supposedly be implemented. Assuming that the argument does extend to type I constructions, it will indicate the

existence of fractional electrically charged states with TeV scale masses, out of which at least one must be stable. The experimental limits on such states are rather strong. From the point of view of string models it will be interesting to learn whether such states are naturally avoided in type I string constructions.

Nevertheless, disregarding theoretical prejudices, from a purely phenomenological perspective, it remains an interesting question whether the idea of TeV scale gravity can bypass all the experimental constraints, at least naively. As shown in ref. [2] the observed small value of Newton's constant at large distances can be ascribed to the spreading of the gravitational force in n "large" extra dimensions. The volume of the extra dimensions is fixed by Gauss law to be

$$r^n \simeq M_{\text{Pl}}^2 / M_*^{n+2}, \quad (1.1)$$

where M_{Pl} is simply related with the Newton coupling constant, $M_{\text{Pl}}^{-2} = 4\pi G_N$. Thus, for $M_* \sim 1\text{TeV}$ already for $n = 2$ the experimental limits on gravitational strength forces are satisfied. This observation prompted a surge of interest in the possibility of TeV scale gravity [3], which investigate the phenomenological viability of this scenario as well as some early attempts to construct viable type I string models. One particularly interesting aspect of the TeV gravity scenario is in regard to light neutrino masses. The reason being that the neutrino sector is precisely the sector that in the TeV scale scenario probes the bulk physics, whereas all the other Standard Model states are confined to the brane.

In ref. [5, 6] the issue of neutrino masses in the TeV gravity scenario was studied. In this paper we examine several phenomenologically related questions. We focus on the scenario suggested in ref. [5] *. The mechanism proposed in ref. [5] assumes a bulk right-handed neutrino. The smallness of the neutrino masses then arises due to the suppression by the volume of the extra dimensions of the couplings of the bulk modes with the branes fields. All interactions of the bulk right-handed neutrino modes with the left-handed neutrino are then suppressed by the volume factor. However, one still has to sum over the tower of Kaluza-Klein modes, with the cut-off imposed at the effective string scale. The interesting case being, of course, $m_s \sim 1\text{TeV}$, which will have dramatic signatures in the coming collider experiments. After summing over the heavy Kaluza-Klein modes one in general gets enhancement of the couplings with potentially interesting consequences for already existent data. Within the context of TeV scale gravity with large extra dimensions, a bulk singlet neutrino is quite interesting from a phenomenological point of view.

In this paper we show that the possible existence of bulk neutrinos impose the constraints on the scale M_* much stronger than those following from gravitational

* The scenario proposed in ref. [6] stipulates Majorana masses for the right-handed neutrino coming from the tower of Kaluza-Klein modes. However, as noted in ref. [5], the mass terms of Kaluza-Klein heavy modes are necessarily Dirac because they originate from higher dimensional kinetic term. Another observation on the mechanism proposed in ref. [6] is that the right-handed component is the light eigenvalue whereas the left-handed neutrino is heavy. The scenario of ref. [6] is therefore not viable phenomenologically and will not be discussed further here.

interactions. Taking the neutrino mass splitting and mixing in the phenomenologically interesting range, we examine the possible implications for experiments in the leptonic sector. Unlike the case with the gravitational interaction, the most restrictive limits on M_* comes from where experimental limits on the lepton nonuniversality and flavor changing processes in the charged lepton sector. An additional important aspect of the derived phenomenological constraints on the cut-off scale M_* is that they are held for even for large number of the extra dimensions.

2 Bulk neutrino masses

We briefly recap the bulk mechanism for generating small neutrino masses. Consider a five dimensional theory with coordinates (x^μ, y) , where $\mu = 0, \dots, 3$ and y compactified on a circle with radius R . One assumes a bulk fermion state, which is a Standard Model singlet, while the lepton and Higgs doublets are confined to the brane. The bulk Dirac spinor is decomposed in the Weyl basis $\Psi = (\nu_R, \bar{\nu}_R^c)$ and takes the usual Fourier expansion

$$\nu_R^{(c)}(x, y) = \sum_n \frac{1}{\sqrt{2\pi r}} \nu_{Rn}^{(c)}(x) e^{iny/r} \quad (2.1)$$

The four dimensional action then contains the usual tower of Kaluza–Klein states with Dirac masses n/r and the free action for the lepton doublet, localized on the wall. We consider here the case in which one assumes exact lepton number conservation, which forbids the Majorana masses, and the leading interaction term between the bulk fermion and the walls fields is

$$S^{\text{int}} = \int d^4x \lambda l(x) h^*(x) \nu_R(x, y=0) \quad (2.2)$$

where λ is a dimensionless parameter. Such a coupling breaks $n+4$ Poincare invariance which is still legitimate because the existence of the wall also breaks it. After compactification, this Dirac field appears on the wall in numerous KK copies. What is more important, however, is that the Yukawa coupling λ is rescaled in the same way as the graviton and dilaton coupling to all brane fields. Being initially of order one, the effective Yukawa is seen from four dimensions as

$$\lambda_{(4)} = \frac{\lambda}{\sqrt{r^n M_*^n}}, \quad (2.3)$$

which leads to very strong suppression of the Dirac mass even for $\lambda \sim 1$:

$$m = \frac{v}{\sqrt{2}} \lambda_{(4)} = \frac{\lambda v}{\sqrt{2}} \frac{M_*}{M_{\text{Pl}}} \simeq \lambda \frac{M_*}{1 \text{ TeV}} \cdot 5 \cdot 10^{-5} \text{ eV}. \quad (2.4)$$

Here v is electroweak v.e.v. This mass parameter appears in all the couplings of left-handed neutrino with KK tower of singlets so that the resulting mass matrix for every neutrino flavor species looks as follows [5]:

$$\mathbf{M} = \begin{pmatrix} m & 0 & 0 & 0 \\ m & 1/r & 0 & 0 \\ m & 0 & 2/r & 0 \\ m & 0 & 0 & 3/r \end{pmatrix} \quad (2.5)$$

In the limit of $m = 0$, mass matrix (2.5) has one zero eigenvalue and standard ladder of KK masses. The left-handed neutrino is decoupled from the KK tower. When m is kept finite, the left-handed neutrino mixes with other states and the mixing angle θ_k between the left-handed neutrino and k -th KK state is given by

$$\theta_k \simeq \frac{mr}{|k|} \quad (2.6)$$

The extreme smallness of $\lambda_{(4)}$, rescaled in the same way as graviton/dilaton coupling with matter, should lead to the suppression of any observable effect. We would like to point out, however, one serious difference between phenomenological consequences of bulk neutrinos and bulk graviton/dilaton fields. All Standard Model processes observed and measured so far in the terrestrial experiments do not involve the emission of gravitons at somewhat feasible level. Any cross section or decay width for a process involving emission/absorption of real gravitons would be suppressed by $1/M_{\text{Pl}}^2$ at least and hence hopelessly small. Similar arguments apply for radiative corrections induced by gravitons in the loop. When the higher-dimensional Planck scale is as low as 1 TeV, there is a chance to see the deviations from Standard Model predictions, because the suppression by four-dimensional Planck scale is partly compensated by large multiplicity of the graviton appearing in a given process with numerous KK copies. When the emission of gravitons is considered, this multiplicity is limited by the maximal kinematically allowed mass of the KK excitation which is of the order of maximal energy transfer (or release) E . For n compactified dimensions this multiplicity factor is $(Er)^n$ and the initial four-dimensional Planck scale suppression is changed for $M_{\text{Pl}}^{-2}(Er)^n \sim E^n/M_*^{n+2}$. This factor drops with n very rapidly. Graviton exchange, including loop corrections, behave differently and the summation over KK modes should be extended and cutoff at virtual energies of the order M_* so that the resulting suppression is just M_*^{-2} . This is what happens, for example with one loop electroweak+graviton exchange correction to the muon decay width where the resulting modification of Fermi constant is of the order $(16\pi^2 M_*^2)^{-1}$. In both possibilities, emission or exchange, current experimental sensitivity does not allow to probe $M_* > 1$ TeV.

The phenomenological implications of bulk neutrinos which mix with the Standard Model left-handed neutrinos (right-handed antineutrinos) is quite different. The main point here is that left-handed neutrinos do take part in the Standard Model processes

and their admixture to KK states may be seen in the low-energy experiments as a tree-level effect. The change of the decay probabilities for muon, tau and pion, negligible in the case of graviton/dilaton emission, may turn out to be significant in the scenario with bulk neutrinos and produce important limits on M_* .

Let us consider, for example, the muon decay. We normalize the decay probability in such a way that in the absence of any right-handed species it is equal to Γ_0 . In the presence of very heavy right-handed neutrino, with Dirac mixing and the customary see-saw mechanism, the content of left-handed neutrino in the light neutrino mass eigenstate is no longer 1, and the resulting probability to decay via W-exchange is $\Gamma_0(1 - \theta^2)$, where $\theta = M_D/M_R$. For one heavy neutrino species this change of the probability is marginal. In the case of the mass matrix (2.5) the change in the probability is given by

$$\Gamma_{\mu \rightarrow e \bar{\nu}_e \nu_\mu} = \Gamma_0 \left(1 - \sum_{KK} \frac{m_1^2 r^2}{k^2} - \sum_{KK} \frac{m_2^2 r^2}{k^2} \right) \quad (2.7)$$

where m_i denotes i -th generation neutrino Dirac mass. The summation over KK states begins at $k_0 \sim (m_\mu r)^2$ and should be cut off at $k_{max} \sim (M_* r)^2$. The exact value for k_0 is unimportant because the sum is totally dominated by the large k . Regardless the fact that every entry in the sums of Eq. (2.7) is very small, after summation the corrections to the width would be described only in terms of the ratio $\lambda_i^2 v^2/M_*^2$ and therefore can be significant if λ_i is large. Eq. (2.7) has simple interpretation. Part of the decay probability is lost due to the admixture to the KK copies of right-handed neutrinos which are kinematically inaccessible for the decay.

Let us recall at this point that the main phenomenological motivation to introduce neutrino masses was to resolve some or all observed neutrino anomalies via possible flavor oscillations, i.e. neutrino mass splitting. In the scenario with bulk neutrinos, flavor splitting, $m_i^2 - m_j^2 \neq 0$, originates from difference in Yukawa couplings and the sums $\sum m_i^2 r^2/k^2$ are different for different i . As a result, muon decay width, tau decay width and tau decay branching ratios should receive different corrections so that the admixture to KK excitations is seen as the effective violation of the lepton universality. On the other hand, the universality of lepton-W couplings is checked experimentally to be precise at 0.3% accuracy level. Therefore we expect that charged pion, muon and tau decay data should provide sufficiently strong limits on r or, equivalently, on M_* .

The resolution of the neutrino anomalies through flavour oscillations requires also non-zero mixing angles among different neutrino species. When loop corrections are considered, mixing angles and splitting of eigenvalues should lead to flavor changing processes in the charged lepton sector. Thus, experimental limits on $\mu \rightarrow e \gamma$, $\tau \rightarrow e \gamma$ decay width and $\mu - e$ conversion provide additional constraints on this scenario.

3 Effective violation of lepton universality

Both $e - \mu$ and $\tau - \mu$ universality is checked at low energies to high degree of precision (See, for example, Ref. [7]). In the case considered, the effective violation of the $e - \mu$ universality due to the admixture to bulk neutrinos can be constrained from charged pion decay, as the Standard Model predictions and the experimental results for $\Gamma(\pi^- \rightarrow e^- \bar{\nu}_e)/\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu)$ coincide rather precisely. In terms of the sum over KK states we have

$$\left| \frac{g_\mu^{eff}}{g_e^{eff}} \right|^2 = \frac{1 - \sum \frac{m_2^2 r^2}{k^2}}{1 - \sum \frac{m_1^2 r^2}{k^2}} \simeq 1 - \sum \frac{(m_2^2 - m_1^2) r^2}{k^2} = 1.003 \pm 0.003. \quad (3.1)$$

The summation over KK modes is generally divergent. It cannot be performed “exactly” and we have to introduce the ultraviolet cutoff $\Lambda \sim M_*$:

$$\sum_{k_1, \dots, k_n}^{|\mathbf{k}| < (\Lambda r)^n} \frac{(m_2^2 - m_1^2) r^2}{k^2} = (m_2^2 - m_1^2) \frac{S_{n-1}}{n-2} \Lambda^{n-2} r^n \simeq \frac{S_{n-1}}{n-2} \frac{(m_2^2 - m_1^2) M_{Pl}^2}{M_*^4} \quad (3.2)$$

Here S_{n-1} is the result of angular integration; the volume of $n - 1$ dimensional sphere. We can see that 4-dimensional Planck scale reappear in the numerator, so that the result can be given in terms of initial non-suppressed Yukawa couplings and fundamental scale M_* . The result is very sensitive to the cutoff parameter Λ , which can be somewhat different from M_* , as it was advocated in refs. [2]. However, the precise knowledge of the cutoff parameter cannot come from the qualitative picture of “brane world” and requires a particular realization of this scenario in a rigorously formulated theory (i.e. string theory) which does not exist at the moment. Requiring that the effective lepton nonuniversality be smaller than the experimental accuracy, we get

$$\frac{S_{n-1}}{n-2} \frac{|\lambda_2^2 - \lambda_1^2| v^2}{2M_*^2} < 3 \cdot 10^{-3}. \quad (3.3)$$

For $\Delta\lambda^2$ of order one, pion decay is sensitive to the scales of order 10 TeV.

For $n = 2$ the sum is logarithmically divergent so that $1 - \sum m^2/m_{KK}^2 \simeq 1 - \pi\lambda^2 v^2/M_*^2 \ln(M_{Pl}/M_*)$. It is clear that the logarithm can, in principle, overcome $\lambda^2 v^2/M_*^2$ suppression and higher order terms in m^2 should be included so that the correct result will be proportional to $(1 + \pi y^2 v^2/M_*^2 \ln(M_{Pl}/M_*))^{-1}$. Numerically, the logarithm is close to 35 which will give ~ 6 factor enhancement in the limits on M_* .

The limits on $\mu - \tau$ universality obtained from μ and τ total widths and τ decay branching ratios are also very stringent; repeating the same arguments we get:

$$\frac{S_{n-1}}{n-2} \frac{|\lambda_3^2 - \lambda_2^2| v^2}{2M_*^2} < 6 \cdot 10^{-3}. \quad (3.4)$$

In the assumption of $\lambda_1, \lambda_2 \ll \lambda_3 \sim 1$ it corresponds to sensitivity to $M_* \simeq 8$ TeV for $n=3$ and $M_* \simeq 30$ TeV for $n=2$.

The exclusion lines on $\sqrt{|\lambda_i^2 - \lambda_j^2|} - M_*$ plane for the case $n = 3$ are given in Figure 1. When at least one of the Yukawa couplings is of order one, low-energy lepton universality constraints push M_* to be larger than $8 - 10$ TeV. One can exclude λ_i from the data and plot the constraints on M_* versus phenomenologically more attractive quantities $m_i^2 - m_j^2$. Logarithmically scaled exclusion plots are given in Figure 2. Numerical smallness of the neutrino masses (2.4) leads also to rather small splittings, unless M_* is very large. Nevertheless, lepton nonuniversality constraints limit M_* quite strongly over the entire domain of the phenomenologically interesting mass splittings (for recent discussions of various possibilities in neutrino sector see Ref. [8]). It is interesting to note that $\mu - e$ universality is sensitive to $M_* \sim 2$ TeV even for $|m_1^2 - m_2^2| \sim 10^{-10} \text{ eV}^2$, which induces vacuum oscillations of neutrinos at the scale comparable with Earth-Sun distance [9].

4 Flavor-changing processes in charged lepton sector

The measurement of possible $\mu \rightarrow e\gamma$ decay puts a very stringent bound on the branching ratio for this processes, $B(\mu \rightarrow e\gamma) < 4.9 \times 10^{-11}$.

The amplitude of the $\mu \rightarrow e\gamma$ transition can be parametrized in the form of the usual dipole-type interaction:

$$\mathcal{M}_{\mu \rightarrow e\gamma} = \frac{1}{2} \bar{e}(d_L P_L + d_R P_R) \sigma^{\alpha\beta} F_{\alpha\beta} \mu \quad (4.1)$$

where $P_{L(R)}$ is left(right)-handed projector. The partial width for this process, reexpressed in terms of d_L and d_R , is:

$$\Gamma_{\mu \rightarrow e\gamma} = \frac{1}{16\pi} (|d_L|^2 + |d_R|^2) m_\mu^3. \quad (4.2)$$

Comparing it with the standard decay width, $\Gamma_{\mu \rightarrow e\nu\bar{\nu}} = \frac{1}{192\pi^3} G_F^2 m_\mu^5$ and using the experimental constraint on the branching ratio, we get the following limit on the dipole amplitude:

$$|d| = \sqrt{(|d_L|^2 + |d_R|^2)/2} < 3.5 \cdot 10^{-26} \text{ e} \cdot \text{cm}. \quad (4.3)$$

Flavor changing dipole amplitude d can be easily computed in our case in terms of lepton and neutrino Yukawa couplings, mixing angles and fundamental scale M_* .

The largest contribution to the amplitude originates from the mixing with heavy KK states, $m_{KK} \gg M_W$. This means that in one-loop diagram the longitudinal part of W -propagator should give the biggest contribution. In the more convenient t'Hooft gauge the leading result comes from charged Higgs exchange. Assuming for

simplicity 2×2 flavor structure, we obtain the dipole amplitude d_R in the following form:

$$|d_R| = \frac{|(\lambda_1^2 - \lambda_2^2) \sin \theta \cos \theta| S_{n-1} m_\mu}{48\pi^2 M_*^2} \begin{cases} (n-2)^{-1} & \text{for } n > 2 \\ \ln M_*^2/M_W^2 & \text{for } n = 2. \end{cases} \quad (4.4)$$

In the summation over KK states we neglected the influence of the lower limit, assuming that $M_W \ll M_*$. The result can be trivially generalized on 3×3 case to include the mixing with τ -neutrino. Comparing d_R with the experimental constraint (4.3), for the case $n = 3$ we deduce the following limit on the combination of couplings, mixing angles and mass scale M_* :

$$|(\lambda_1^2 - \lambda_2^2) \sin \theta \cos \theta| \left(\frac{1 \text{ TeV}}{M_*} \right)^2 < 1.1 \cdot 10^{-3} \quad (4.5)$$

Reexpressed in terms of neutrino masses, this constraint takes the form:

$$\frac{|(m_1^2 - m_2^2) \sin(2\theta)|}{10^{-5} \text{ eV}^2} \left(\frac{1 \text{ TeV}}{M_*} \right)^4 < 5.6 \cdot 10^{-7}. \quad (4.6)$$

We see that the $\mu \rightarrow e\gamma$ decay is extremely sensitive to the scenario with bulk neutrinos if the intergenerational mixing angles are large. For $n = 3$, $\theta \sim \pi/4$ and the splitting of order 10^{-5} eV^2 , $\mu \rightarrow e\gamma$ decay probes M_* as high as 35 TeV. For $n = 2$, this decay is sensitive to M_* of order 100 TeV. In the “just so” neutrino scenario, corresponding to the case of large mixing angle and extremely small mass splitting of order 10^{-10} eV^2 , $\mu \rightarrow e\gamma$ branching ratio limits M_* to be heavier than 2 TeV. We note also that this calculation can be performed directly in the coordinate space with the divergent integral cut at the fundamental length scale M_*^{-1} . In this way the proportionality of the result to $(\lambda_1^2 - \lambda_2^2)/M_*^2$ is even more explicit than in the calculation performed in the momentum representation.

Another important FCNC effect, where significant experimental progress is plausible, is $\mu - e$ conversion. In the scenario with bulk neutrinos it is predominantly generated by the following effective interaction

$$\mathcal{L}_{int} = \kappa J_\beta^{(q)} \bar{e} \gamma_\beta (1 - \gamma_5) \mu, \quad (4.7)$$

where $J_\lambda^{(q)} = \frac{2}{3} \bar{u} \gamma_\lambda u - \frac{2}{3} \bar{d} \gamma_\lambda d$. The coefficient in front of this operator can be calculated similarly to $\mu \rightarrow e\gamma$ amplitude. In the result for κ we keep only the contributions enhanced by large logarithmic factor, $\ln(M_*^2/M_W^2)$, which simplifies the calculation:

$$\kappa = \frac{\alpha}{6} \frac{|\cos \theta \sin \theta (\lambda_1^2 - \lambda_2^2)|}{M_*^2} \ln \frac{M_*^2}{M_W^2}. \quad (4.8)$$

The experiment limits the isoscalar part of the vector interaction [10],

$$g_V < 3.9 \cdot 10^{-7} \quad (4.9)$$

with g_V and κ being simply related by $g_V G/\sqrt{2} = \kappa/3$. Combining the experimental result and Eq. (4.8), we obtain the following constraint on M_* , the splitting of Yukawa couplings and mixing angle:

$$|(\lambda_1^2 - \lambda_2^2) \sin \theta \cos \theta| \frac{\ln(M_*^2/M_W^2)}{7} \left(\frac{1\text{TeV}}{M_*} \right)^2 < 1.4 \cdot 10^{-3}. \quad (4.10)$$

Comparing it with the limit (4.5), we conclude that $\mu \rightarrow e\gamma$ branching ratio and $\mu - e$ conversion provide comparable limits on M_* . The constraints coming from $\tau \rightarrow e\gamma$ are not competitive with (4.5) and (4.10).

5 Conclusions

In this paper we examined some of the implications of the recently proposed mechanism for generating small neutrino masses in the TeV scale gravity scenario. Similar to the suppression of the gravitational interaction, the small neutrino masses are obtained due to the suppression of the effective Yukawa coupling between the wall left-handed neutrinos and the bulk-right handed neutrinos by the volume of the compactified dimensions. We have shown that the intergenerational mass splitting and mixing, naturally brought about by neutrino Yukawa matrix, leads to the observable effects at low energies. The mixing of the left-handed neutrino with heavy KK modes, different for each flavor, alters the decay widths of the charged pion, muon, tau and tau branching ratios. This is in contrast to the case of the graviton emission which brings only marginal change of these decay widths. Considering the effects on lepton-universality; and flavor changing transitions we showed that the experimental data constrains the higher dimensional Planck scale to be of the order $M_* \geq 10\text{TeV}$, over most of the interesting range of neutrino mass splitting and therefore outside the reach of the LHC. In the absence of concrete models it is in general found that it is difficult to further constrain the TeV gravity scenario. On the other hand, we feel that it is important to assert that gravity at the TeV scale necessarily disallows the traditional Grand Unification paradigm, and will necessarily imply new avenues that have been previously unforeseen.

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Figure Captions

Figure 1. The exclusion plot for M_* vs. $\lambda_{ij} \equiv \sqrt{|\lambda_1^2 - \lambda_2^2|}$ in the case of $n = 3$.

Figure 2. Logarithmically scaled exclusion plot for M_* vs. $m_{ij}^2 \equiv |m_i^2 - m_j^2|$ in the case of $n = 3$.

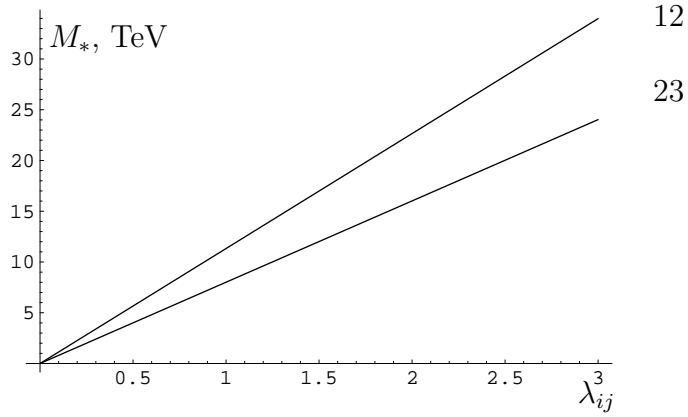


Figure 1

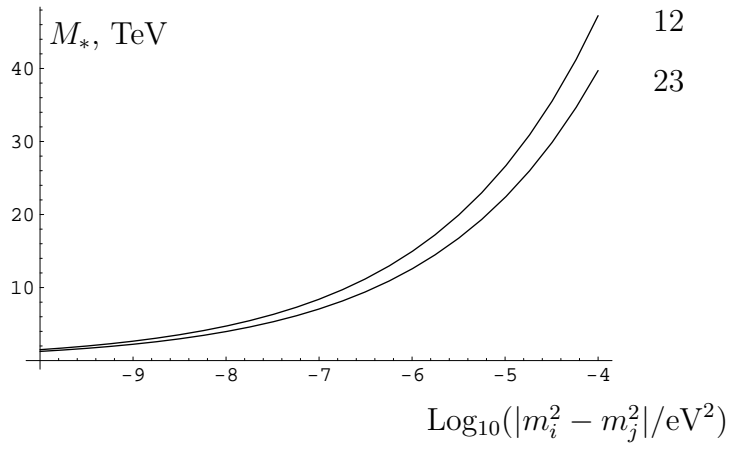


Figure 2